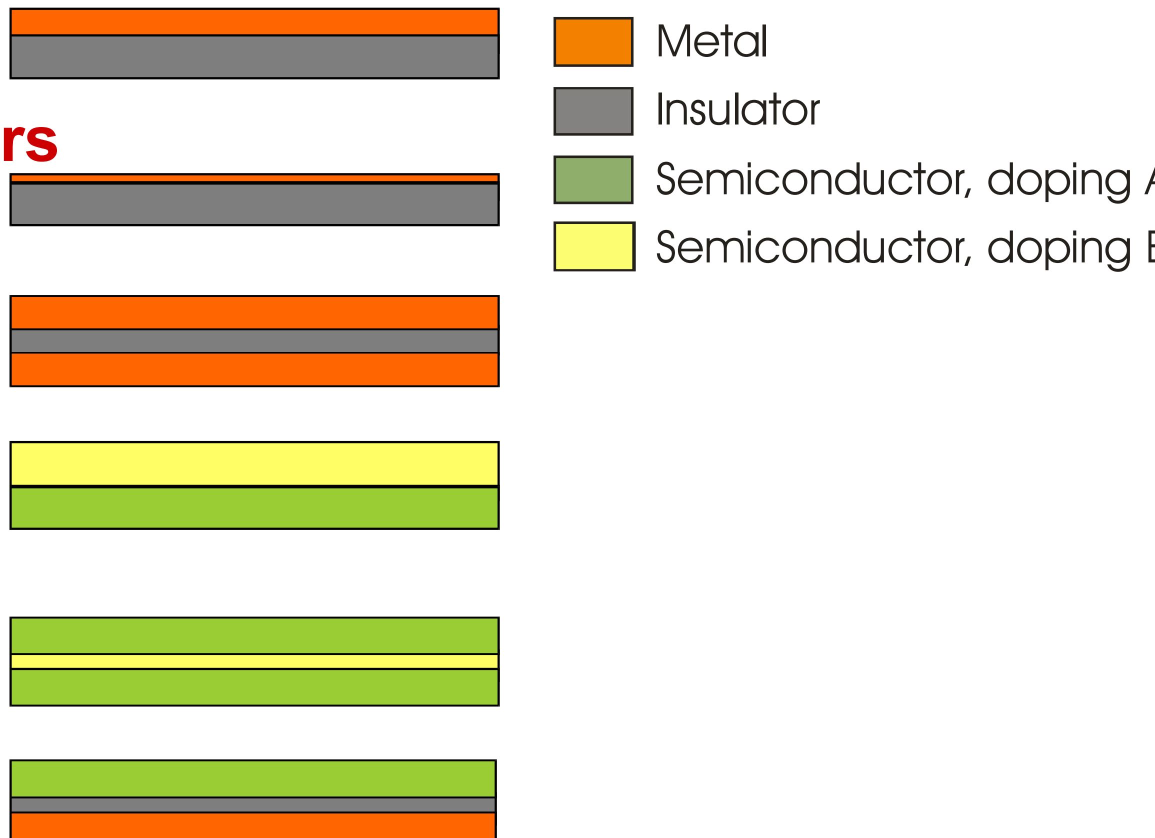
Repetition: Electronic Components

By thin film technology the following electronic components can be realized:

+ **Interconnect**
+ **Thin film resistors**
+ **Condensers**
+ **Diodes**
+ **Transistors**
+ **MOSFETS**



Repetition: Theory of Conductivity

Drude theory:

$$\vec{j} = \frac{ne^2\tau}{2m_e}\vec{E} = \sigma \cdot \vec{E}$$

j = Current density
E = E-field
σ = Conductivity
n = Number of charges
e = Elementary charge
m_e = Electron mass
τ = Mean collision time

The central point of the Drude theory is the

Mean collision time τ

Repetition: Conductivity and Transport Theory

For a mathematically correct and also for a quantum mechanically sound calculation of the conductivity of solid bodiesor thin films Boltzmann's transport theory has to be applied.

Repetition: Conductivity and Current Density

General approach for the calculation of the conductivity in metals:

Starting point: current density

$$\vec{j}_e = -ne\vec{v} = -e\vec{v}\int_N \frac{dn}{V} = -e\vec{v}\frac{N}{v} = -ne\vec{v}$$

$$dn = 2 \cdot f_0(E) \cdot d\Phi$$

$$f_0(E) = \frac{1}{1+e^{(E-E_f/k_B T)}}$$

Fermi-distribution

$$d\Phi = \frac{d^3x d^3p}{h^3}$$

Phase space volume, number of states in the phase space volume element $d^3x d^3p$

2 Spin-number

Repetition: the Boltzmann Equation

Descriptions of changes in f by collisions

$$\frac{df(\vec{r},\vec{v},t)}{dt} = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Formulation for charged particles

$$\frac{\partial f}{\partial t} + \vec{\nabla}_r f \underbrace{\frac{d\vec{r}}{dt}}_{v} + \vec{\nabla}_v f \underbrace{\frac{d\vec{v}}{dt}}_{a=\vec{F}/m=-\frac{e\vec{E}}{m}} = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

$$\frac{\partial f}{\partial t} + \vec{v}\vec{\nabla}_r f - \frac{e\vec{E}}{m}\vec{\nabla}_v f = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Repetition: Drude Model/Transport Theory

Drude:

$$\vec{j} = \frac{ne^2\tau}{2m_e}\vec{E} = \sigma \cdot \vec{E}$$

Boltzmann:

$$\vec{j} = \frac{8\pi e^2 \tau m_e{}^2 v_f^3}{3h^3}\vec{E} = \sigma\vec{E}$$

$$n = \frac{1}{V}\int dn =$$

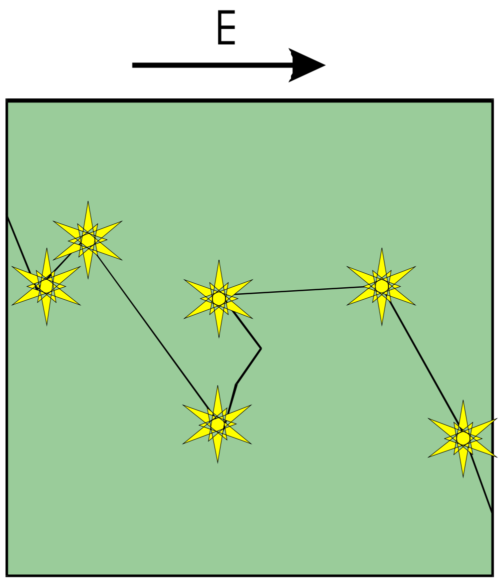
$$= \frac{1}{V}\int_{\vec{R},\vec{v}} 2f_0\left(\frac{m}{h}\right)^3 d^3x\,d^3v = \frac{8\pi}{3}\left(\frac{v_f m}{h}\right)^3$$

$$\vec{j} = \frac{ne^2\tau}{m_e}\vec{E} = \sigma \cdot \vec{E}$$

$j_{Drude}=1/2 j_{Boltzmann}$

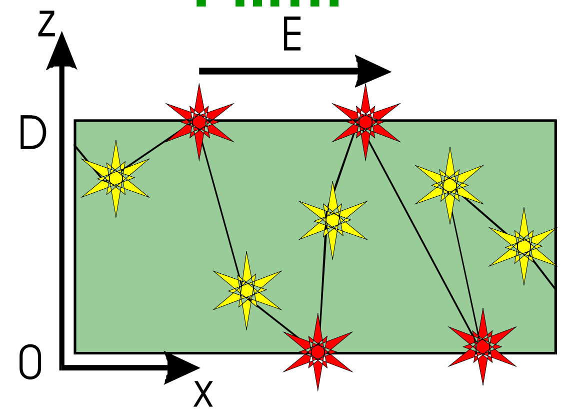
Repetition: Thin Films

Bulk:



$$\frac{1}{\tau} = \frac{1}{\tau_G} + \frac{1}{\tau_K} + \frac{1}{\tau_V}$$

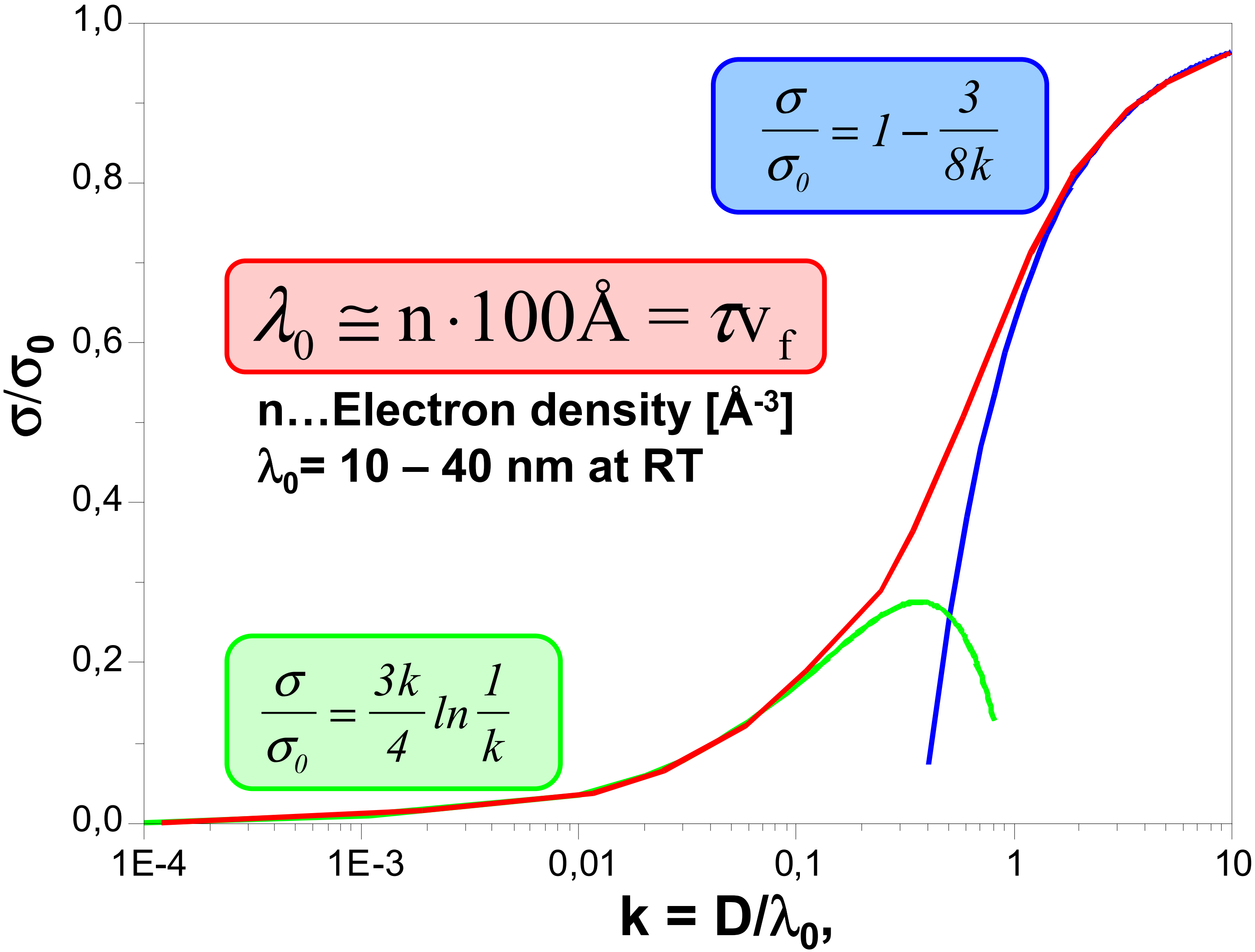
Film:



$$\frac{1}{\tau} = \frac{1}{\tau_G} + \frac{1}{\tau_K} + \frac{1}{\tau_V} + \frac{1}{\tau_I}$$

The Interfaces at z=0 and z=D are additional scattering centers for electrons!

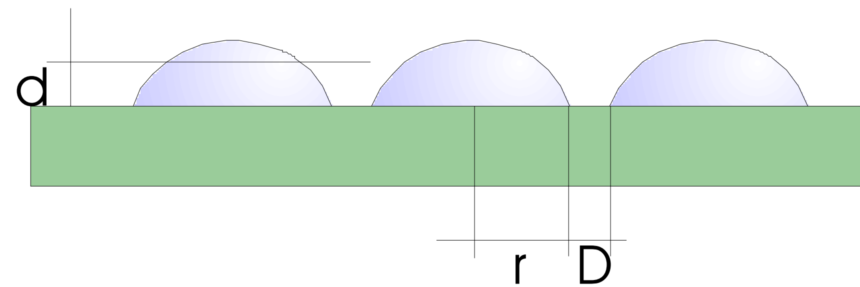
Repetition: Conductivity Approximations



Repetition: Real Thin Film Systems

Ideal:

Real:



Experiment:

$$\sigma_{dis} << \sigma_{kont}$$

$$\sigma \propto e^{-A/k_B T}$$

$$\sigma = \sigma(E)$$

$$\sigma = \sigma(r, D)$$

Justification:

Diskontinuity
Thermionic emission
Field emission
Tunnel effect

Optical Properties

Fundamentals:

The optical properties of materials result from the reaction of the electronic system to electromagnetic fields.

Static:

$$\vec{j} = \sigma \cdot \vec{E}$$

Dynamic:

General:

$$\ddot{\vec{u}} + \Gamma\dot{\vec{u}} + \omega_0^2\vec{u} = \frac{Q}{m}\vec{E}(\vec{u}, t)$$

Perpendicular, plane wave, z=0:

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = -\frac{e}{m}E_0 \exp(i\omega t)$$

Dielectric Function

The dielectric function ε describes the "response" of a system of electrons with densities n_n, chatacteristic frequencues ω_{0n} and damping constants Γ_n to a incoming signal:

Perpendicularily impinging, plane wave:

$$\varepsilon(\omega) = \varepsilon_0 [1 + \chi(\omega)] = 1 + \frac{e^2}{m} \cdot \sum_n \frac{n_n}{\omega_{0n}^2 - \omega^2 - i\omega\Gamma_n}$$

$\chi(\omega)$…electrical susceptibility

Dielectric Function and Conductivity

There is a connection between dielectric function and conductivity which allows a discrimination between metals and insulators at vanishing frequencies ω:

$$\varepsilon(\omega) = \varepsilon_0 + \frac{i\sigma(\omega)}{\omega}$$

$\omega \rightarrow 0$:

Metals:

$\sigma(\omega)$ **is finite** $\rightarrow$ $\varepsilon(\omega)$ **diverges**

Insulators:

$\sigma(\omega)$ **vanishes** $\rightarrow$ $\varepsilon(\omega)$ **remains finite**

In the case of high frequencies ω metals and insulators behave the same!

Dielectric Function and Refractive Index

With the help of the dielectric function it is possible to define the refractive index of a material as tzhe sum of a real and an imaginary part.

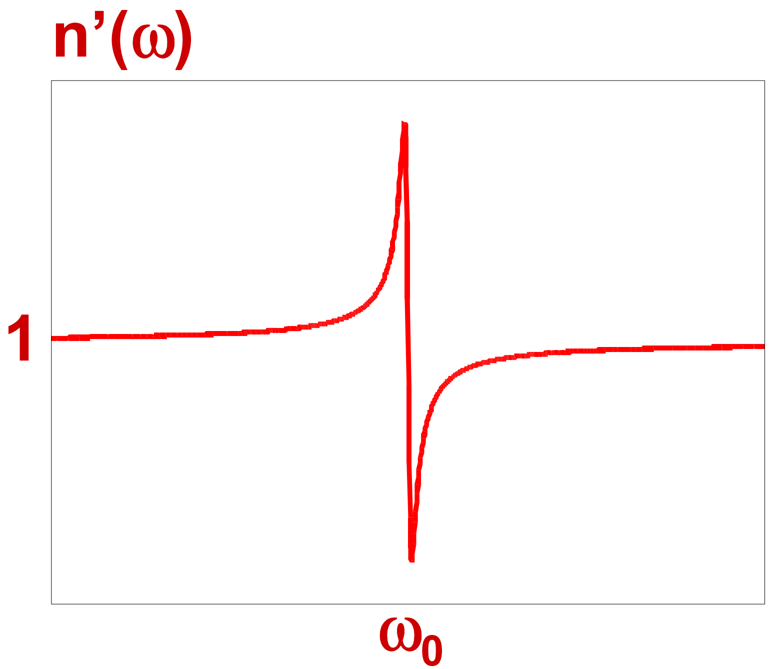
One atom species:

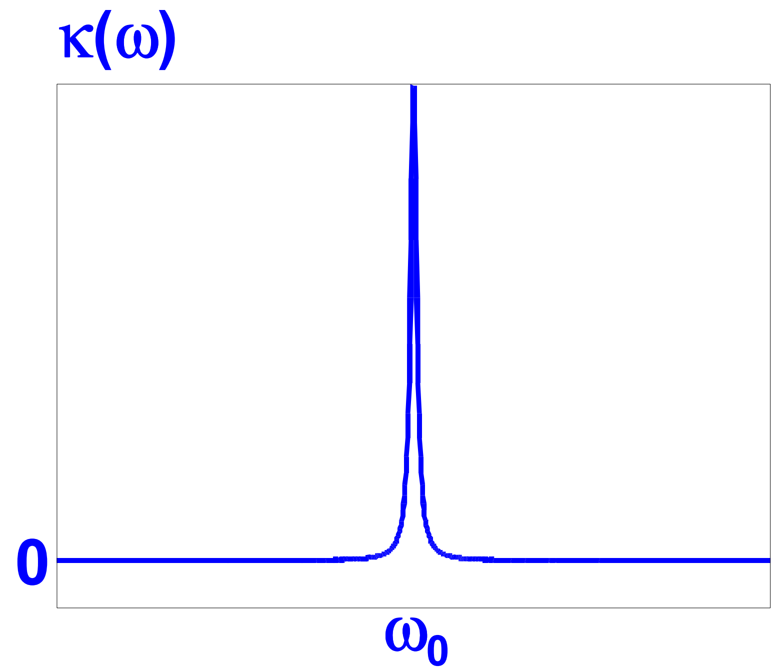
$$n(\omega) = n'(\omega) + i\kappa(\omega)$$

$$n'(\omega) = 1 + \frac{ne^2}{\varepsilon_0 m} \cdot \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2}$$

$$\kappa(\omega) = \frac{ne^2}{\varepsilon_0 m} \cdot \frac{\omega\Gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2}$$

Real and Imaginary Refractive Index

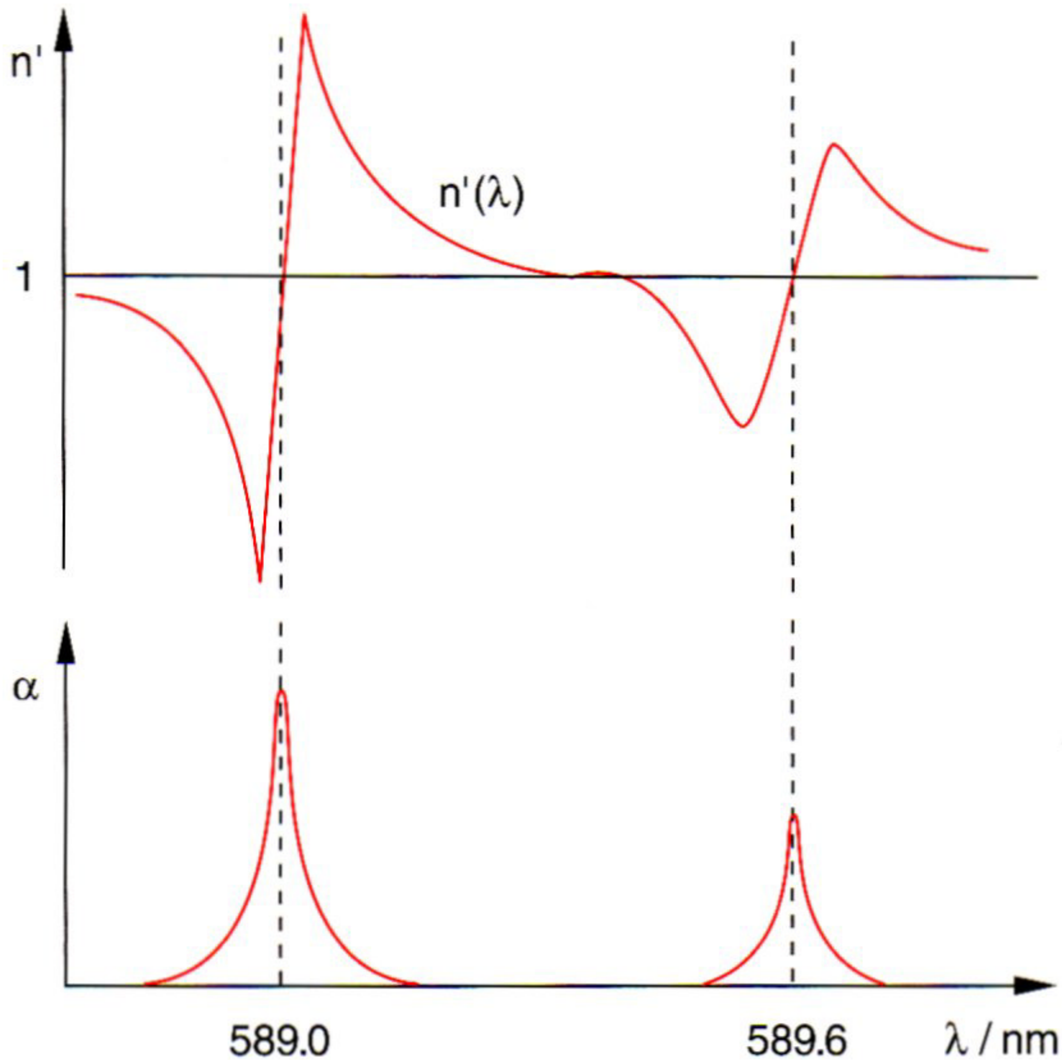




The real part of the refractive index corresponds to refractive index n, as it appears in Snellius law of refraction.

The imaginary part corresponds to the absorption of energy in the medium.

Example: Na Double Line



Optics – Frequency Independent Refraction

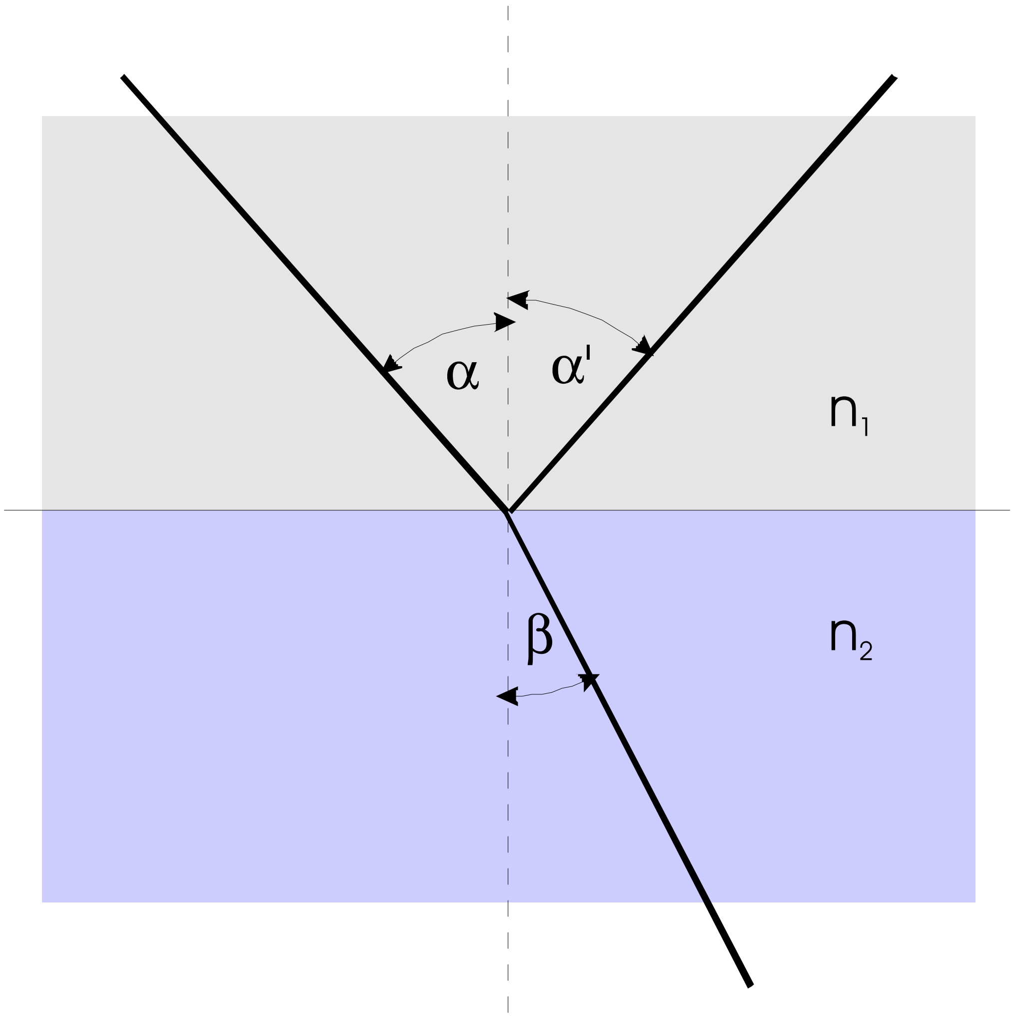
For optics the following conservation law is valid:

$$T + R + A + S = 1$$

T … Transmission
R … Reflection
A … Absorption
S … Scattering

For geometric optics the refraction index n can be considered as frequency independent.

Optics – Interfaces



Reflection:

$$\alpha = \alpha'$$

Refraction:

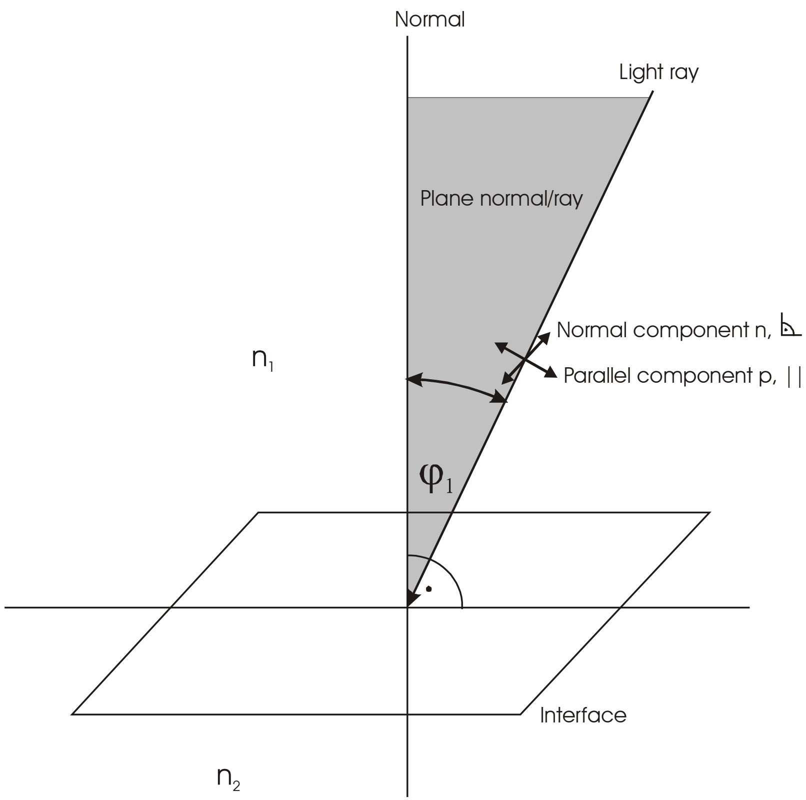
$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

Wavelength:

$$\lambda_i = \frac{\lambda_{Vak}}{n_i}$$

Fresnel's Equations

Most general description of the passage of a light ray (plane electromagnetic wave) through an interface



Reflection:

$$r_k^p = \frac{n_{k-1}\cos\varphi_k - n_k\cos\varphi_{k-1}}{n_{k-1}\cos\varphi_k + n_k\cos\varphi_{k-1}}$$

$$r_k^n = \frac{n_{k-1}\cos\varphi_{k-1} - n_k\cos\varphi_k}{n_{k-1}\cos\varphi_{k-1} + n_k\cos\varphi_k}$$

Transmission:

$$t_k^p = \frac{2n_k\cos\varphi_k}{n_{k-1}\cos\varphi_k + n_k\cos\varphi_{k-1}}$$

$$t_k^n = \frac{2n_k\cos\varphi_k}{n_k\cos\varphi_k + n_{k-1}\cos\varphi_{k-1}}$$

Fresnel's Equations: Simplification

2 Media, indices of refraction n_1, n_2, perpendicular impingement, i. e.: $\varphi_1 = \varphi_2 = 0°$

Reflection:

$$r_k^p = r_k^n = \frac{n_1 - n_2}{n_1 + n_2}$$

Transmission:

$$t_k^p = t_k^n = \frac{2n_1}{n_1 + n_2}$$

Optical Film Thickness

Electromagnetic radiation passes from Vacuum into a Medium with refractive index n:

Frequency ω:

$$\omega_n = \omega_{Vak}$$

Wavelength λ:

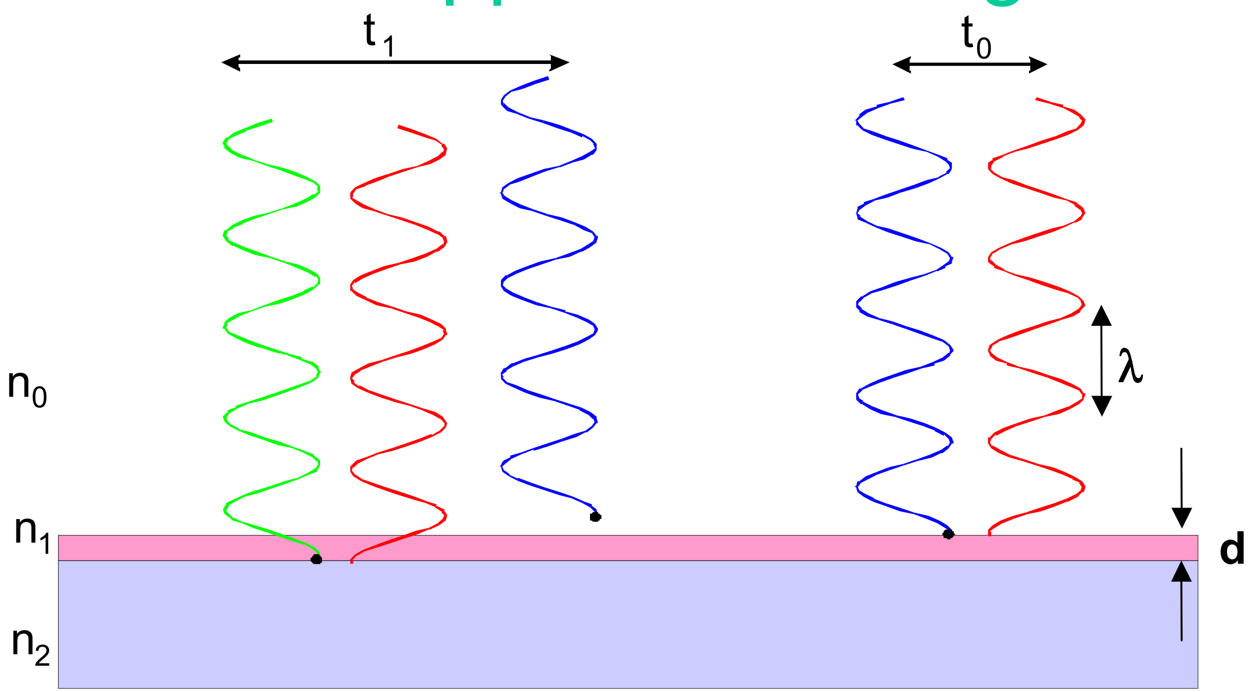
$$\lambda_n = \frac{\lambda_{Vak}}{n}$$

If a film thickness is given as the moltiple of a wavelength, λ_n is meant. This film thickness is called “optical film thickness", d_{opt}.

It is:

$$d_{Opt} = n \cdot d$$

Reflection Suppression: Single Reflection I



$n_0 < n_1$

$n_1 < n_2$

$d = \lambda/4$

Amplitudes of reflected radiation:

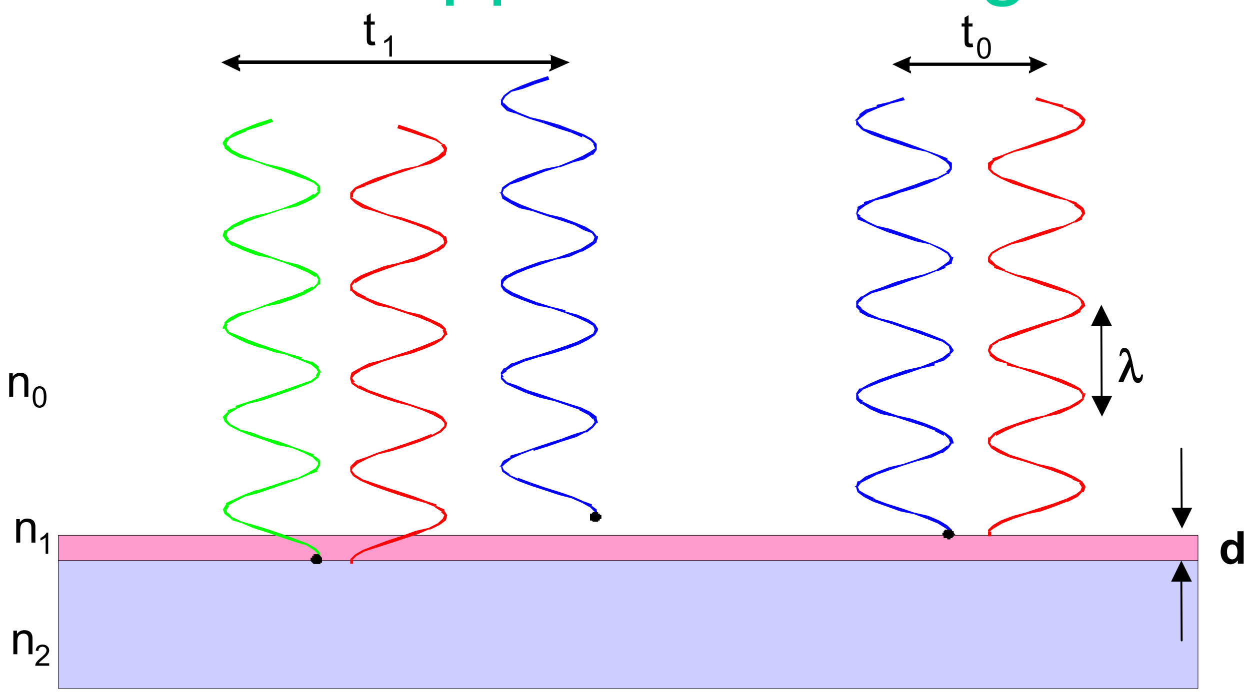
$$n_1 < n_2 : \quad r = \frac{n_1 - n_2}{n_1 + n_2} < 0$$

Phase jump: π

$$n_1 > n_2 : \quad r = \frac{n_1 - n_2}{n_1 + n_2} > 0$$

Phase jump: 0

Reflection Suppression: Single Reflection II



$n_0 < n_1$

$n_1 > n_2$

$d = \lambda/4$

Intensities of reflected radiation:

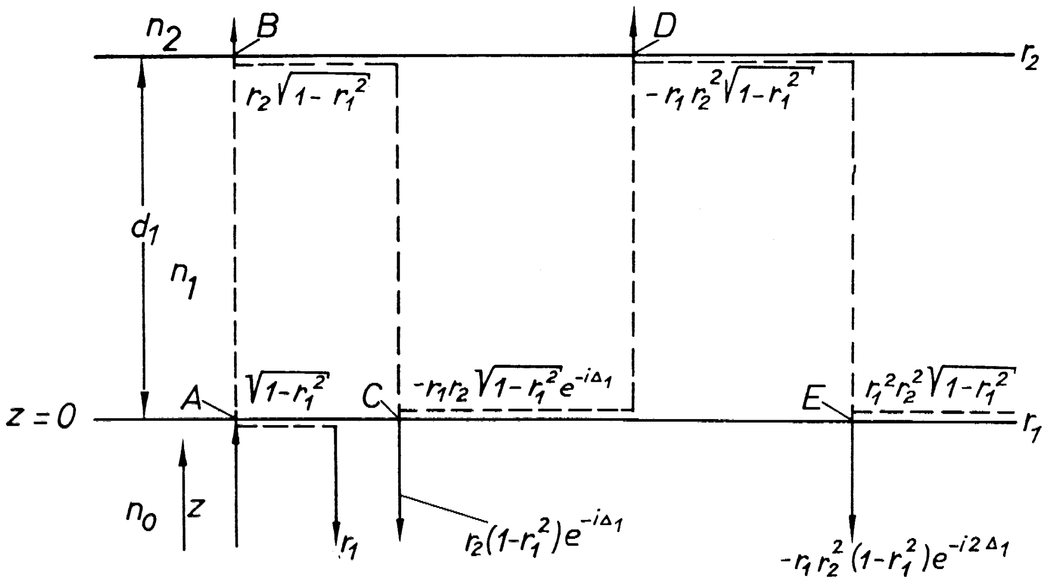
$$I_r^0 = \frac{(n_0 - n_1)^2}{(n_0 + n_1)^2} \qquad I_r^2 = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$$I_r^0 = I_r^2 \Rightarrow n_1 = \sqrt{n_0 \cdot n_2}$$

Amplitude requirement

Single Layer: Multiple Reflections

Situation sketch:



Summing up the partial beams:

$$r = \frac{r_1 + r_2 e^{-i2\delta_1}}{1 + r_1 r_2 e^{-i2\delta_1}}$$

$$t = \frac{t_1 t_2 e^{-i2\delta_1}}{1 + r_1 r_2 e^{-i2\delta_1}}$$

$$\delta_1 = \frac{2\pi}{\lambda} \cdot n_1 \cdot d_1 \cdot \cos\varphi_1$$

Thickness dependent phase-shift of the electromagnetic wave after the wave enters the coating.

Real Transmission and Reflection

$$R = \frac{r_1^2 + 2r_1 r_2 \cos 2\delta_1 + r_2^2}{1 + 2r_1 r_2 \cos 2\delta_1 + r_1^2 r_2^2}$$

$$T = \frac{n_0}{n_2} \frac{t_1^2 t_2^2}{1 + 2r_1 r_2 \cos 2\delta_1 + r_1^2 r_2^2}$$

$$r_{1,2} = \frac{r_{1,2}^p + r_{1,2}^t}{2}$$

$$t_{1,2} = \frac{t_{1,2}^p + t_{1,2}^t}{2}$$

$$\delta_1 = \frac{2\pi}{\lambda} \cdot n_1 \cdot d_1 \cdot \cos \varphi_1$$

Amplitude and Phase Requirement

There is a null in the reflected intensity, if the folluwing requirements are fulfilled:

$$n_1 = \sqrt{n_0 n_2}$$

Amplitude requirement

$$\delta_1 = (2m-1)\frac{\pi}{2}$$

$$m = 1,2,3,\ldots$$

Phase requirement

With this tere is:

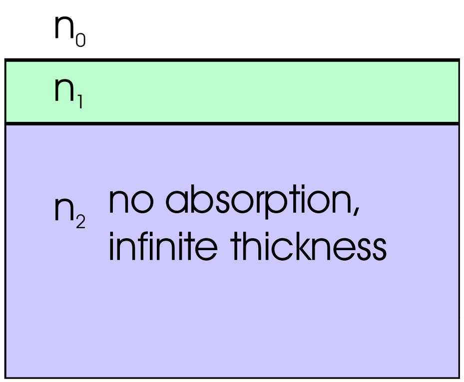
$$R_{min} = \left(\frac{n_1^2 - n_0 n_2}{n_1^2 + n_0 n_2}\right)^2$$

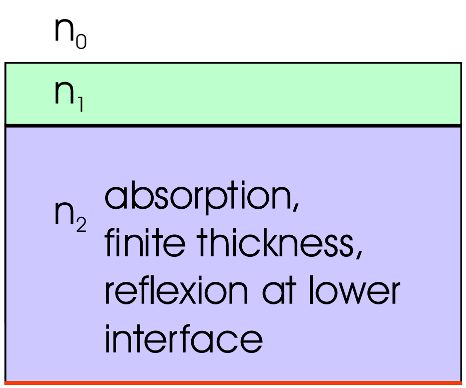
$$\lambda_{min} = \frac{4 n_1 d_1}{2m-1} \Rightarrow d_1^{min} = \frac{2m-1}{4 n_1}\lambda$$

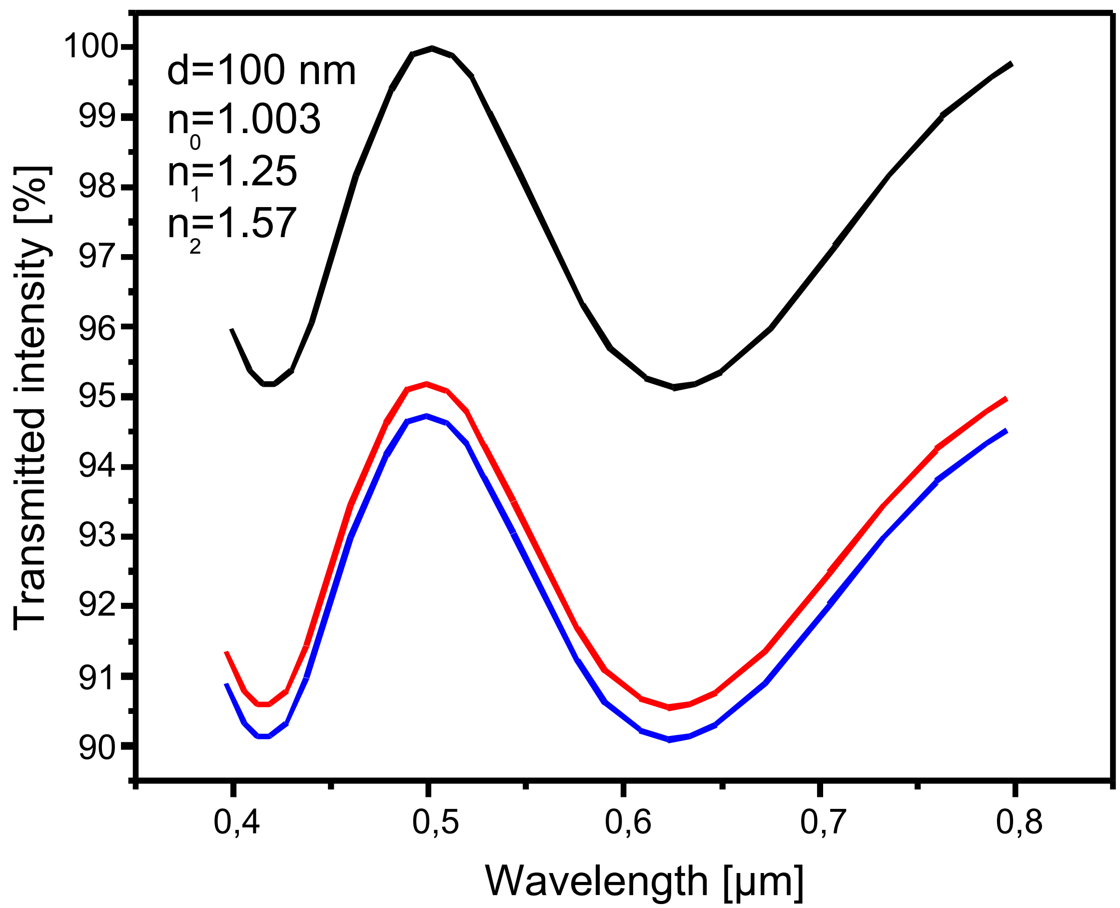
$$m = 1 : d_1^{min} = \frac{2-1}{4 n_1}\lambda = \frac{1}{4 n_1}\lambda$$

"λ/4 " - Schicht

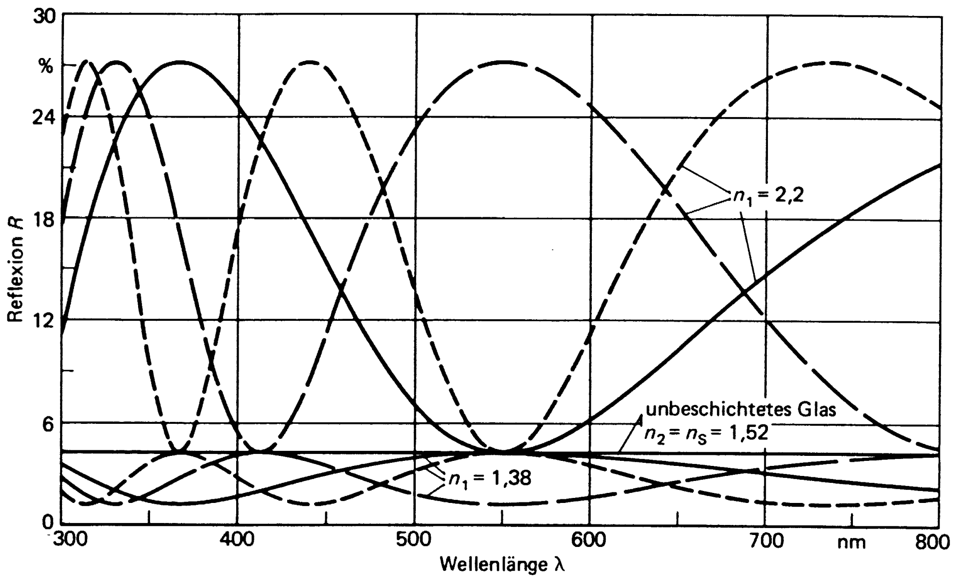
Ideal und Real Thin Film System



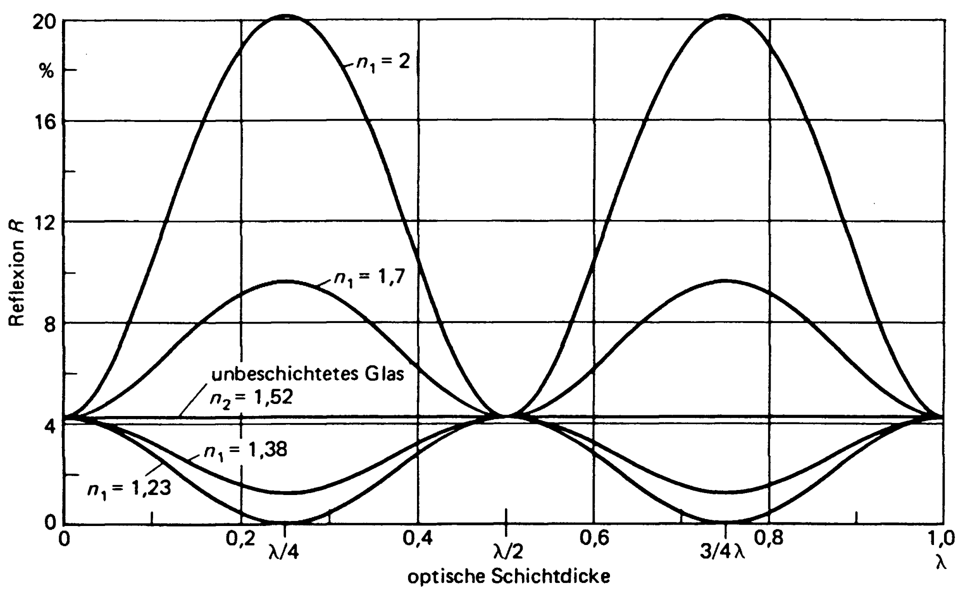




Single Coating - General



Reflection/Wavelength



Reflection/Thickness

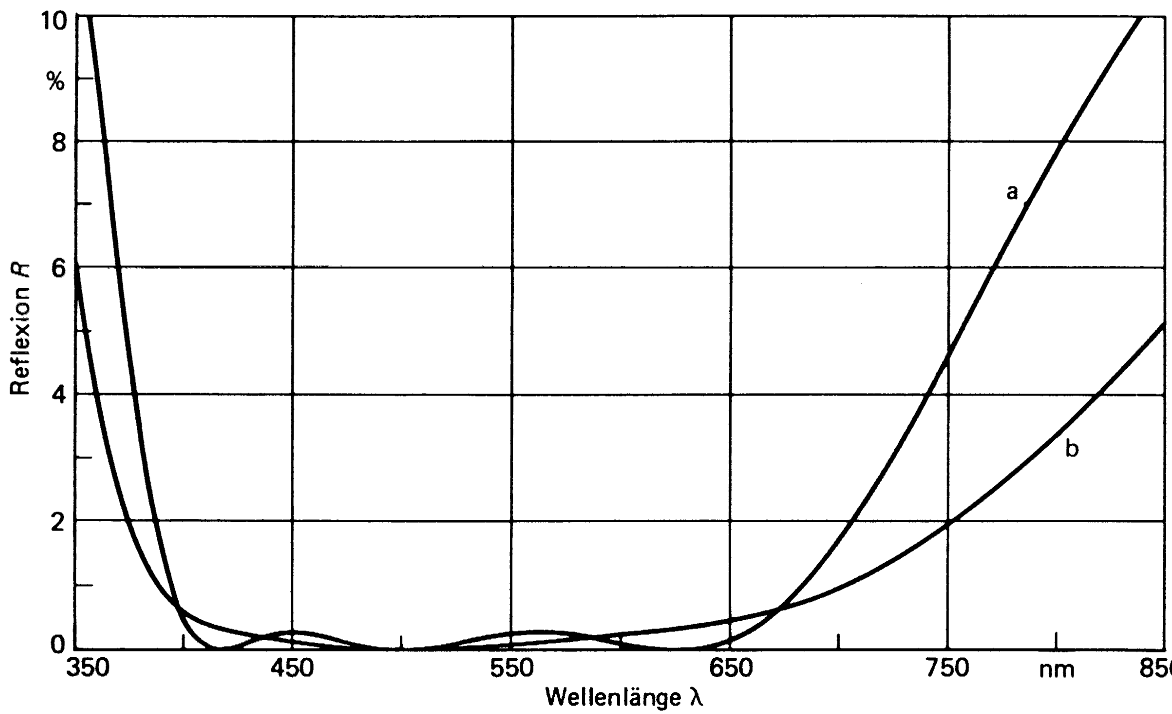
Multi Coating - Reflection Suppression

Single coatings do not allow for reflection suppression in a broad range of wavelengths.

➔Application of systems consisting of multiple coatings

With multiple coating systems it is possible to generate reflection suppressing coatings in a wide spectral range on substrates with low indices of refraction (1,5-1,7).

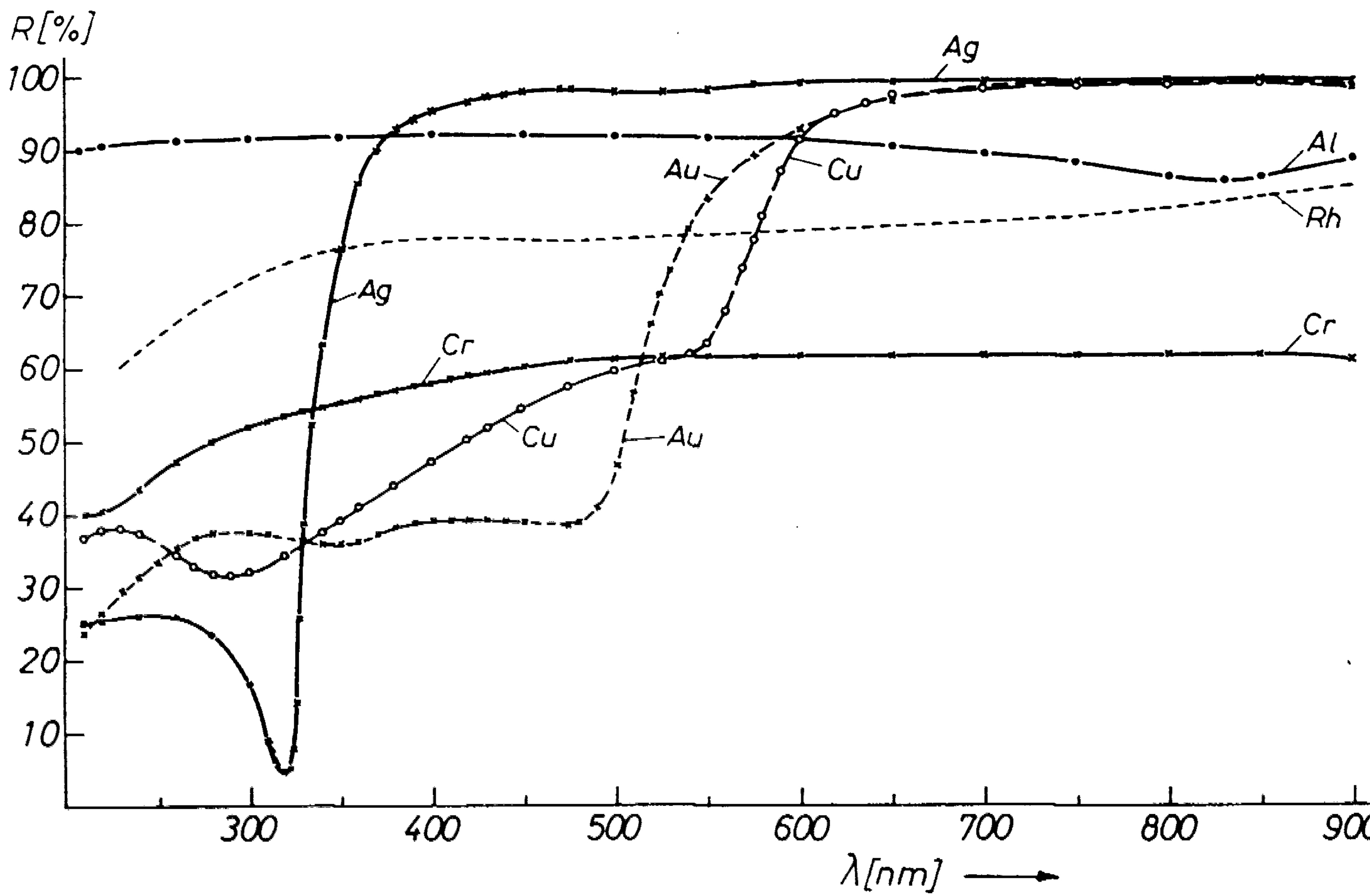
Multiple Coatings - Example



a) λ/4 - λ/2 - 3λ/4
b) λ/4 - λ/2 - λ/4

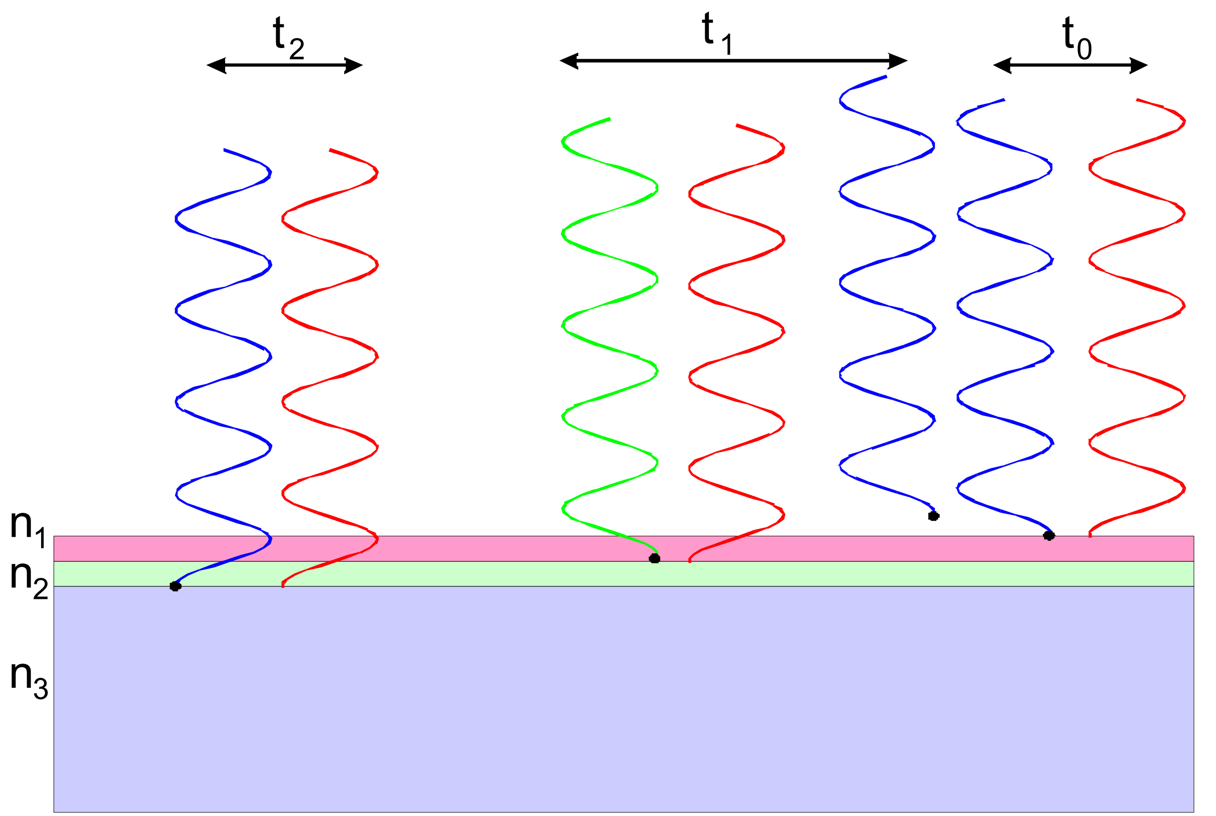
$n_0 = 1$
$n_1 = 1.38$
$n_2 = 2.1$
$n_3 = 1.7$
$n_4 = 1.52$

Reflection Enhancement: Metal Films



Reflection Enhancement: Dielectric Mirror I

System consisting of λ/4-coatings with alternating indices of refraction:



$n_1 < n_2$

$n_2 > n_3$

Reflection Enhancement: Dielectric Mirror II

A dielectric mirror consists of a multilayer made from λ/4-coatings with alternating high (H) and low (L) indices of refraction.

Anzahl der Schichten	Reflexion in %	
	$n_L = 1{,}38$ $n_H = 2{,}3$ $n_s = 1{,}51$	$n_L = 1{,}47$ $n_H = 2{,}3$ $n_s = 1{,}51$
3	53,89	53,23
5	85,20	80,84
7	94,67	92,15
9	98,08	96,79
11	99,31	98,68
13	99,75	99,46
15	99,91	99,78
17	99,97	99,91
19	99,99	99,63

Further Optical Elements

With the thin film systems discussed so far it is possible to manufacture filter systems with almost arbitrary properties.

The basic elements are:

- **Reflection suppressing coatings**
- **Interference coatings**
- **Metal mirrors**
- **Dielectric mirrors**
- **Nanocluster (color centers and pigments)**

These elements allow the realization of many optical systems.