

# ACCURATE MEASUREMENT OF THE MAGNETOSTRICTION OF SOFT MAGNETIC MATERIALS

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## Abstract

A survey of different measuring methods suited for soft magnetic materials is given. The methods are subdivided into direct and indirect methods. The SAMR method is best suited especially for low magnetostrictive ribbons. Experimental examples will be shown.

## 1. Introduction

In soft magnetic materials the magnetostriction is an important parameter determining the domain structure, the magnetization process and consequently the coercivity field. For very soft magnetic materials (low coercivity, high permeability) the magnetostriction  $\lambda_s$  has to be low whereas a high value of  $\lambda_s$  is interesting for the application of such a material for stress or strain sensors. The magnitude and the sign of the magnetoelastic energy determines the stress dependence of the hysteresis loop, which can be used for the sensor function. The magnetostriction is an intrinsic property of magnetic materials, which is for a crystalline material a direction dependent tensor  $\lambda_{hkl}$  which is mainly caused by the orbital coupling of the magnetic moment. From the temperature and concentration dependence of  $\lambda_s$  conclusions about the corresponding mechanism are possible. Therefore the accurate determination of the magnetostriction is very important.

The magnetostriction can be measured by direct and indirect methods. Direct methods are for example measurements with strain gauges, capacitance transducers or interferometers. For crystalline materials the use of strain gauges is most common. The most sensitive method is the capacitance method. All these methods have the disadvantage that they require a special sample preparation and they do not deliver the "true" saturation magnetostriction. Strain gauges are easy to handle but limited in sensitivity. Additionally the saturation magnetostriction constant has to be determined from measurements parallel and perpendicular to the external field.

Indirect methods are the Becker-Kersten method [1] and the SAMR method [2] (Small angle magnetization rotation). For soft magnetic ribbons because of the special geometry beside the stress dependence of the hysteresis the SAMR method delivers very accurate results. This method can also be used from 4.2 K up to high temperatures. With the Becker-Kersten method the magnetostriction is determined from the stress dependence of the hysteresis loop. An overview about these methods can be found in reference [3]. A comparison of some available methods is presented in Table 1.

*Table 1 Comparison of some methods to measure magnetostriction*

Method	Manner	Sensitivity	Remark
Strain gauges	Direct	$\pm 1 \times 10^{-6}$	Determine $I_{//}$ and $I_{\perp}$ independently, difficult to perform at high temperature
Capacitance	Direct	$\sim \pm 10^{-10}$	Most sensitive method but also difficult to carry out at high temperature
Stress dependence of hysteresis loop	Indirect	$\sim \pm 10^{-8}$	Good for negative and very small value of $I_s$

SAMR (Small Angle Magnetisation Rotation)	Indirect	$\sim \pm 10^{-9}$	High sensitivity, suitable for ribbons, usable at all temperature to determine $\mathbf{I}_s$
Transverse susceptibility	Indirect	$\sim \pm 10^{-9}$	Variation of SARM method

## 2 Experimental methods

There are several ways to determine the magnetostriction using *direct* as well as *indirect methods* see e.g. the review of Lachowics in the Proceedings of the SMM Conference in Eger 1984 [4] and e.g. [5]. The actual selection of a certain measuring technique depends on the sensitivity, on the temperature range and also the sample geometry.

### 2.1 The saturation magnetostriction

The saturation magnetostriction constant  $\lambda_s$  characterizes the change in a linear dimension of a magnetic material in an external field and is in general a material tensor. The number of elements depends on the symmetry of the crystal structure. For isotropic materials the relative change in length depends on the angle between the magnetization vector and the strain. Therefore usually one takes an extrapolation from the strain of the high field range to zero and measures the values of elongation or contraction along the z-axis by applying an H-field parallel or perpendicular to z:  $\lambda = 3/2 \lambda_s (\cos^2 \theta - 1/3)$ , see fig.1.

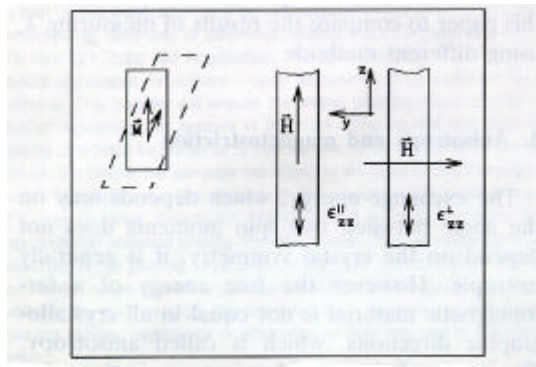


Fig.1: Definition of the saturation magnetostriction measurement

The free energy of a ferromagnetic sample is not equal in the crystallographic directions. This is called anisotropy. The magnetostrictive distortion is determined by the elongation derivative from the stress dependence of the anisotropy energy. The value of  $\lambda_s$  can be obtained by extrapolating the strains from the high field range to zero field:  $\lambda_s = 2/3 (\lambda_{||} - \lambda_{\perp})$ . The factor (here 2/3) depends on the symmetry of the lattice. In the case of some texture this can lead to wrong  $\lambda_s$  values. Generally follows from this expression that (nearly) with all direct methods one has to determine both values:  $\lambda_{||} - \lambda_{\perp}$ .

Stress dependence: By taking the series expansion of the anisotropy and there the higher orders one realize that  $\lambda_s$  is determined by the stress dependent anisotropy and even the stress dependent magnetostriction itself:  $\lambda_s = 2/3 K_1 - 4/3 K_2 \sigma$  with the different anisotropy constants.

### 2.2 Strain gauges

Strain gauges consist of a thin meander shaped film where the resistance changes due to a change of length. This change of resistance can be measured using a bridge circuit. In Fig.2 the general set up is shown.

For crystalline materials the use of strain gauges for measuring the thermal expansion or the magnetostriction is most common since their development in 1947 (but very crucial: the applied strain gauges must be larger than the mean crystalline grains!). Usually full or half bridges are applied to compensate temperature changes during measurement. Application of dummies reduces also the influence of temperature and of magnetic field changes during measurements. If an ac-bridge is used the applied frequency limits dynamic measurements! The sensitivity is in the range of  $1 \times 10^{-6}$  running experiments from 4.2 K up to about 200°C depending on the sensor and the glue applied.

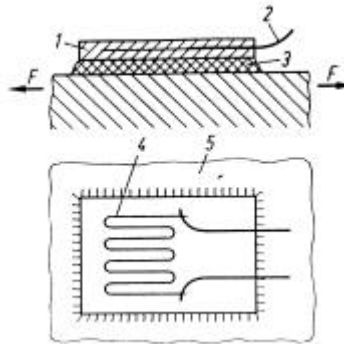


Fig.2. Strain gauge 1 substrate, 2 wires, 3 glue, 4 meander, 5 sample.

The strain gauge is glued on the sample, the wires are connected with a resistance bridge. The change of the resistance is proportional to the local elongation  $\varepsilon$ :

$$\Delta R/R = k\varepsilon = (k/E)\sigma \quad (1)$$

The k-factor gives the sensitivity; it is for metallic strain gauges close to 2. At low temperatures and at high fields deviations due to the magnetoresistance of the material can occur.

The field influence can be reduced applying a second strain gauge (on a nonmagnetic sample) in the neighboring part of the bridge. In this case the magnetoresistive but also thermal contributions are cancelled. Using the strain gauge method for measuring the magnetostriction of a polycrystalline sample needs to measure parallel and perpendicular to the field axis.

### 2.3 Direct displacement method

A *direct* way is the measurement of a displacement caused by the external magnetic field performed by mechanical or optical methods as e.g. micrometer screw, dilatometers or reflection of a laserbeam or interferometry reaching a sensitivity of about  $10^{-11}$ . One sophisticated example is the method of a tunneling tip strain detector by Brizzolara 1990 [6] similar to a scanning tunneling microscope reaching a sensitivity of about  $10^{-9}$  (see fig.3). In all these cases one can measure of course the magnetostriction itself, but also, very important for actuators, the elongation/ contraction as a function of the magnetic field in absolute values!

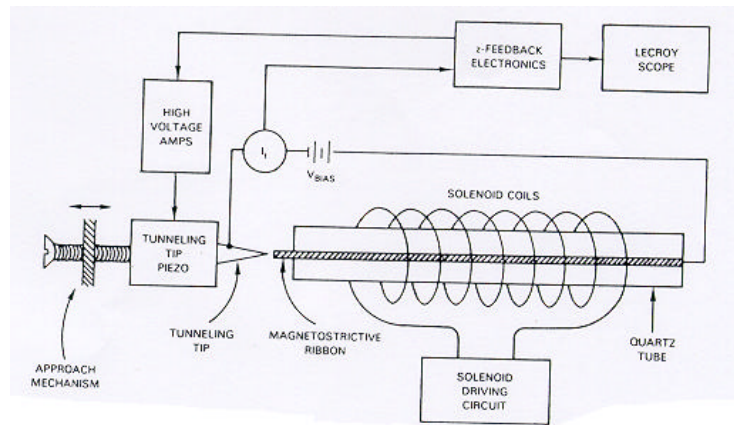
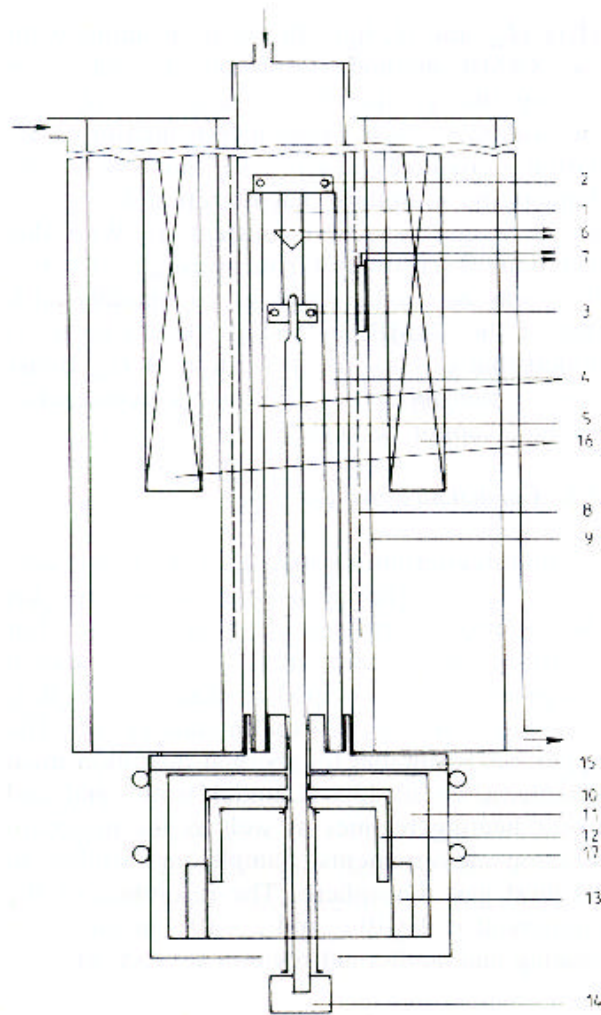


Fig.3: Schematic diagram of the tunneling tip method for measuring the magnetostriction and the displacement values itself.

The most sensitive method is the capacitance method based on the principle of 1920 "ultramicrometer" and can be applied in a wide temperature range, see fig.4 [5], [7]. The so-called cantilever- capacitance method is adapted to thin films [8] 1964 and can be used by measuring the value of  $\Delta C$  over a changing resonance frequency. Also in this case one can measure the magnetostriction, but also, very important for actuators, the elongation/ contraction as a function of the applied magnetic field in absolute values! The sensitivity of the capacitor method can be up to  $10^{-8}$ , however due to the dimensions of the set-up one measures usually only  $\lambda_{||}$ . Therefore the determination of the physically interesting  $\lambda_s$  is not possible. Only if single crystals are available one can determine the elongation in certain crystallographic direction however only parallel to the external field. Fig.4 shows a variation of the capacitance method suited for measuring the thermal expansion and the magnetostriction of amorphous ribbons [9]. In this case the change of length of the sample (1) is transmitted over a mechanical system (3,5) to a condenser set-up (13,14). With a sensitive ac-bridge the change of capacitance  $\Delta C$  which is proportional to the change of length  $\Delta l$  can be determined.



Schematic drawing of the equipment for thermal dilatation and magnetostriction measurements: 1: sample; 2: upper clamp; 3: lower clamp; 4: silica rod; 5: silica tube; 6: thermocouple; 7: platinum resistor; 8: heating coil; 9: ceramic shield; 10: electrostatic shield; 11: insulator; 12: mobile cylindrical condenser electrode; 13: fixed cylindrical condenser electrode; 14: zero position adjustment and weight holder; 15: flange; 16: magnetic field coil; 17: condenser heating coil.

Fig.4: Principle of a capacitance measuring method of long ribbons.

Of course x-ray diffraction experiments determining the field dependence of the lattice constants is a direct way, however due to the long measuring time this method is seldom used. Here the use of high intense synchrotron radiation may open new possibilities.

The measurement of the magnetostriction coefficients on sufficient large single crystals using strain gauges is a very successful method, but for many technical relevant samples good single crystals are not available.

## 2.4 Indirect methods

Usually *indirect methods* are applied utilizing a well known dependence of a physical quantity on strain or displacement due to the magnetostriction or the inverse way (Villari effect) in which a particular magnetic parameter are a mathematical function of the stress applied.

The strain modulated ferromagnetic resonance was introduced 1963 [12] and successfully applied to garnets and spinel ferrites. Films and ribbons can be examined and values of  $\lambda_s$  near  $10^{-9}$  are detectable, but it is restricted to thin samples because of the high frequency based low skin depth.

For soft magnetic ribbons because of the special geometry the SAMR method delivers very accurate results. This method can also be used from 4.2 K up to high temperatures. It was originally developed 1969 by Konishi and adopted by Narita 1980 [2]. Hernando reached values of  $\lambda_s$  near  $10^{-9}$  [10], [11]. Some sophisticated, but related methods are also in use [4], [5] as e.g. the stress dependencies of the transverse susceptibility (by Kraus 1989 [14]): an aligning static field  $H_c$  is applied perpendicular to the ribbon axis and additionally an external stress is applied. The problem with this method is the unfavorable demagnetizing factor. More simple is the stress dependence of the hysteresis loop (Becker Kersten 1930 [1]) which works very well especially for materials with  $\lambda_s < 0$ . Another possibility is to measure the effect or rotation on the magnetization due to twisting a ribbon or the effect of the field dependence of the sound velocity of a magnetoelastic wave. These methods are mainly restricted to negative values of  $\lambda_s$ .

### 2.4.1 Stress dependent hysteresis loop

The very old, but simple and interesting method was first described by Becker, Kersten 1930 and uses the static hysteresis loop by applying different longitudinal stresses, see fig.5. It is easy to handle for ribbons with negative and slightly positive magnetostriction values.

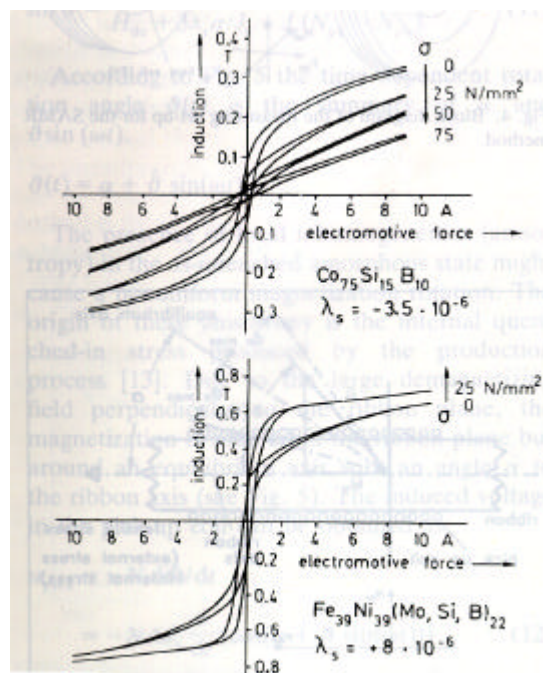


Fig.5: Static hysteresis loop of different amorphous ribbons under external stress

In the case of negative magnetostriction the vector of magnetization is rotated out of the ribbon axis, thus the field  $H_s$  being necessary to saturate the materials will increase with increasing external stress. Minimizing the free energy  $F = E_{stat} + E_{el} = \mu_0 M_s H_s \cos\theta + 3/2 \lambda_s \sigma \sin^2\theta$  with respect to the angle leads to  $\lambda_s = -1/3 \mu_0 M_{sd} dH_s / d\sigma$ . For the measurements with this method the law of approach to saturation has to be considered too. See fig.6: The applied field to reach 90% of saturation is measured as a function of stress:

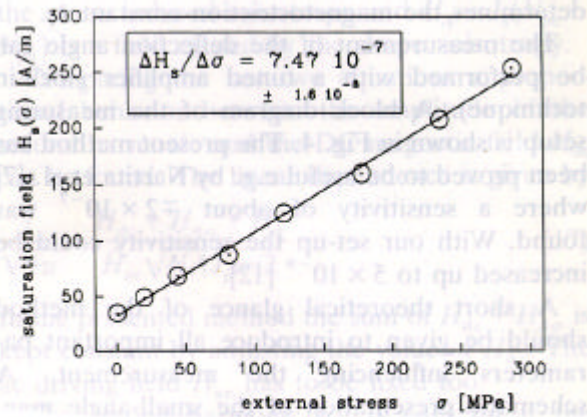


Fig.6: Required field to reach 90% of saturation as a function of external longitudinal stress for an as cast ribbon of  $\text{Co}_{67}\text{Fe}_7(\text{MoSiB})_{29}$ .

## 2.4.2 The principle of the SAMR

In Fig.7 the principal set-up of the SAMR method is shown. A small angle (a few degrees) magnetization rotation in the ribbon plane is caused by a small AC field  $H_{ac}$  perpendicular to the axis. A constant DC field  $H_{dc}$  aligns the magnetic moments. Additionally a stress  $\sigma$  is applied on the ribbon. The total energy is in this case:

$$F = -H_{dc} J_s \cos\theta - H_{ac} J_s \sin\theta + 3/2 \lambda_s \sigma \sin^2\theta + 1/2 J_s^2 (N_{pa} \cos^2\theta + N_{pe} \sin^2\theta) \quad (2)$$

leading to an expression for the value of the deflection angle where  $\theta(t) = \alpha + \theta_0 \sin(\omega t)$ , coming from local inhomogenities as quenched in stresses. The definition of the angles in the ribbon plane is shown in Fig. 7. The induced measuring signal in the pickup system can mathematical be separated into the 1. and 2. harmonics, where the fundamental one is extremely small and usually constant depending only on the inhomogenities and can be eliminated by the lockin amplifier, see fig.8, the block diagram of the SAMR-method. The 2. harmonic contains the information about the magnetostriction. The amplitude of the second harmonics has to kept constant as a function of the stress by tuning the corresponding DC-field.

The deflection angle is detected by a pickup system wound around the ribbon. Under applied stress the DC field is changed as long as the pickup signal is the same. The application of a lockin amplifier is useful in order to suppress the fundamental wave as well as to improve the signal to noise ratio. A detailed calculation of the method is found in [3] starting from the total energy consisting of magnetostatic, magnetoelastic energy and a term with the demagnetizing factors (shape anisotropy). Minimizing the free energy  $F$  leads finally to an expression for the saturation magnetostriction:

$$\lambda_s = -\frac{1}{3} J_s \left( \frac{\Delta H_{dc}}{\Delta \sigma} \right) \quad (3)$$

Because the area of ribbons is often not simple to determine it is senseful for the ribbon geometry to replace the saturation polarization by the direct measurable flux  $\Phi$  and the stress by the force  $F$ .

$$\lambda_s = -\frac{1}{3} \frac{\Phi_{\text{sat}}}{A_{\text{rib}}} \left( \frac{\Delta H_{\text{dc}}}{\Delta F/A_{\text{rib}}} \right) = -\frac{1}{3} \Phi_{\text{sat}} \left( \frac{\Delta H_{\text{dc}}}{\Delta F} \right) \quad (4)$$

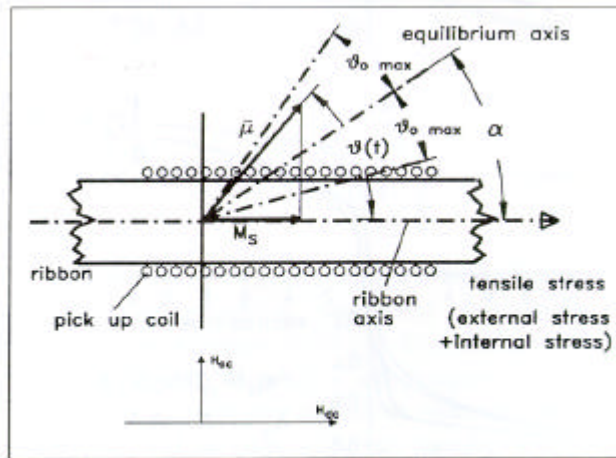


Fig.7: Schematic presentation of the SAMR-Method, the rotation of the magnetization in the ribbon plane.

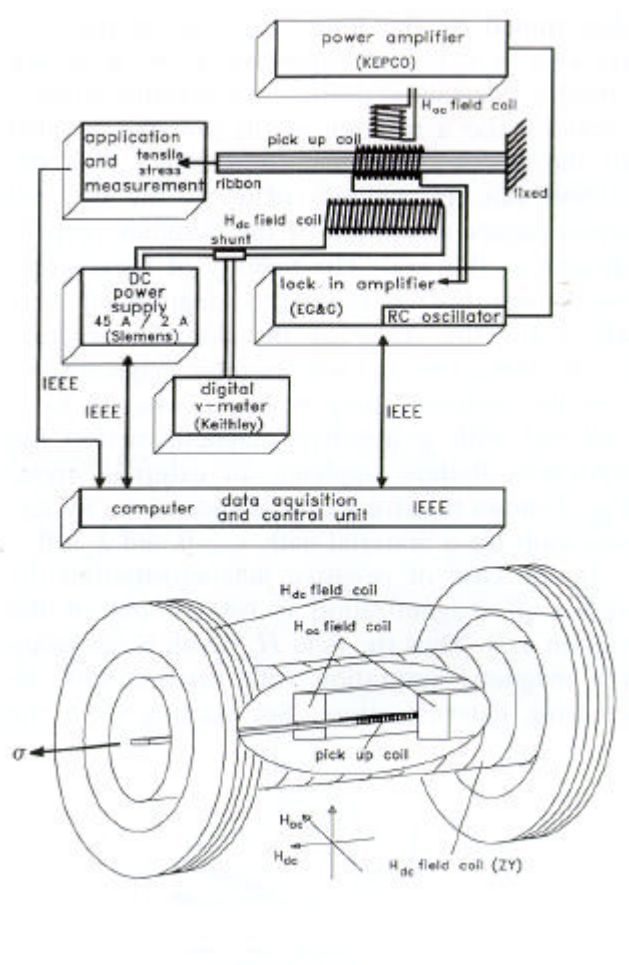


Fig.8: Block diagram of the SAMR-method

The SAMR method is based on the assumption that  $H_{\text{dc}}$  saturates the sample. This is not always the case - therefore it should be mentioned that even soft magnetic materials are



not easy to saturate, this means that the law of approach to saturation not only describes the high field magnetization but also the value of the magnetostriction, as can be seen in the following picture 9:

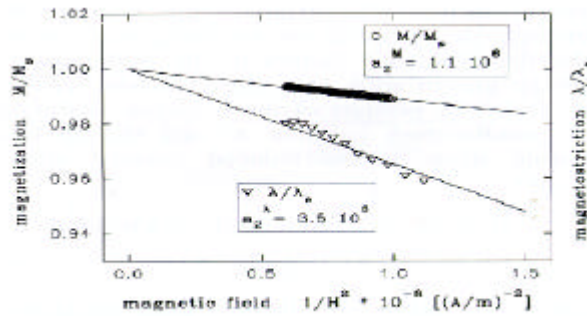


Fig.9: Magnetostriction ( $\nabla$ ) and magnetization ( $\circ$ ) as a function of  $1/H^2$  of amorphous  $\text{Fe}_{76}\text{Si}_{12}\text{B}_{12}$  in the as cast state. The line represents a linear fit with a fit constant  $a_2$ .

The determination of the saturation magnetostriction becomes much more sophisticated, when the value is extremely small, as can be the case in Co-rich  $\lambda_s$ -zero ribbons, where a stress dependence of the magnetostriction can be detected.

- *Stress dependence of the magnetostriction:* The stress dependence of the magnetostriction occurs mainly at high external stresses. This effect is usually only detectable on small magnetostrictive alloys ( $\lambda_s < 10^{-7}$ ). An example for the stress dependence of the magnetostriction is shown in Fig.10.

In the case of a stress dependent magnetostriction the measured values  $H_{dc}(\sigma)$  will not show a linear behavior. An additional quadratic term is used to describe the stress dependent effects:

$$H_{dc} = H_0 + k \cdot \sigma + \kappa \cdot \sigma^2 \quad (5)$$

The fitting procedure delivers the parameters  $k$  and  $\kappa$  and then the magnetostriction value and its stress dependence can be determined:

$$\lambda_s(\sigma) = -\frac{1}{3} J_s \left( \frac{dH_{dc}}{d\sigma} \right) = -\frac{1}{3} J_s \cdot k - \frac{2}{3} J_s \cdot \kappa \cdot \sigma \quad (6)$$

The magnetostriction at zero stress is given by:

$$\lambda_s(\sigma = 0) = -\frac{1}{3} J_s \cdot k \quad (7)$$

The stress dependence of the magnetostriction is calculated by:

$$\frac{d\lambda_s}{d\sigma} = -\frac{2}{3} J_s \cdot \kappa \quad (8)$$

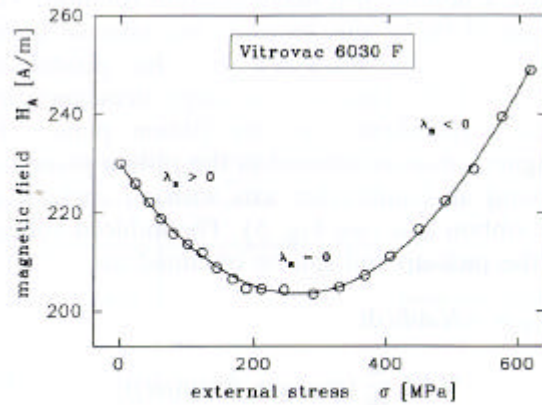


Fig.10: The stress dependence of the anisotropy field  $H_A$  which is proportional to the magnetostriction of a Vitrovac 6030F ribbon with extremely small magnetostriction constant.

### 2.4.3 The transverse susceptibility

This method is similar to the SAMR, it was developed by Kraus [14] but has only a changed geometry. The symbols are equivalent to the SAMR, but only the demagnetizing field has to be considered in detail. Also there from the magnetoelastic contribution the saturation magnetostriction constant is determined.

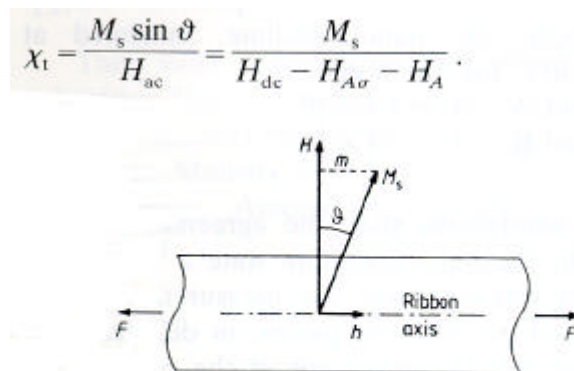


Fig.11: Schematic presentation of the transverse susceptibility method. The ribbon plane coincides with the plane of the figure.

## 3 Experimental example and analysis

In this section as an example selected results and an analysis of the temperature dependence of the magnetostriction as measured on amorphous  $Fe_{85-x}Co_xB_{15}$  is given. Fig. 12 shows the temperature dependence of the magnetostriction in the low temperature range. First of all a phenomenological description of the magnetostriction is given. The magnetostriction shows nearly a linear behavior with temperature. Note that this linear behavior does not exclude that the magnetostriction scales with  $M_s^2(T)$  or  $M_s^3(T)$ . It can be shown that especially in the high temperature range a linear behavior follows the theory. A better description of its low temperature dependence is given by a  $T^{3/2}$  law.

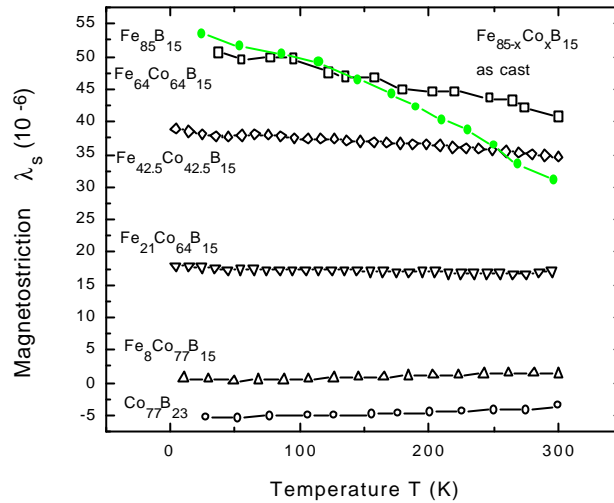


Fig.12. Temperature dependence of magnetostriction in the low temperature range

Fig. 13 shows that the slope of the magnetostriction is higher when the magnetostriction itself is high. Also the magnetostriction depends on the temperature dependence of the magnetization which is in the low temperature range related to the spin wave stiffness constant. Based on these assumptions a corresponding fit was tried which describes the magnetostriction behavior:

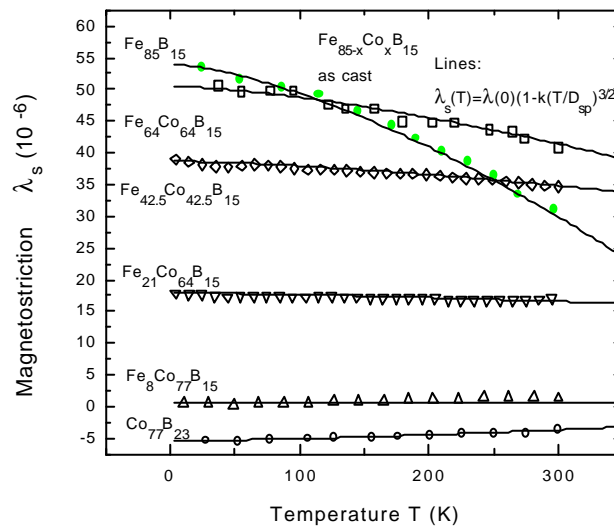


Fig.13. Temperature dependence of magnetostriction in the low temperature range and theory (line)

In a first order approximation assuming a *one ion* behavior in the *low temperature* range the magnetostriction is proportional to  $M_s^3$ , whereas assuming a *two ion* origin corresponds to a  $M_s^2$  proportionality:

$$\begin{array}{c} \text{One-ion model} \\ \lambda_s \propto M_s^3 \end{array}$$

$$\begin{array}{c} \text{Two-ion model} \\ \lambda_s \propto M_s^2 \end{array}$$

The magnetization below room temperature can be very well described by Bloch's law, so one can combine these two theoretical suggestions:

$$\left( \frac{M_s(T)}{M_s(0)} \right)^3 = \left( 1 - \left( \frac{T}{T_0} \right)^{3/2} \right)^3 \cong \left( \frac{M_s(T)}{M_s(0)} \right)^2 = \left( 1 - \left( \frac{T}{T_0} \right)^{3/2} \right)^2 \cong$$

Generally the values of  $T_0$  can be quite high (between 1000-3000 K). Therefore the ratio of  $T/T_0$  is quite small and one can make an approximation and rewrite the equations:

$$1 - 3\left(\frac{T}{T_0}\right)^{3/2} + \dots = 1 - 3k_1\left(\frac{T}{D_{sp}}\right)^{3/2} \quad \left| \quad 1 - 2\left(\frac{T}{T_0}\right)^{3/2} + \dots = 1 - 2k_1\left(\frac{T}{D_{sp}}\right)^{3/2}$$

where  $D_{sp}$  is the spin wave stiffness constant and the factor  $k_1$  results from the theory of the spin waves. The exact expression is given by:

$$k_1 = \frac{0.1174\mu_B k_B^{3/2}}{\mu_0 M(0)}$$

Depending on the used units for the spin wave constant  $D_{sp}$  one obtains for this factor:

$$k_1 = \frac{7.019 \cdot 10^{-65}}{\mu_0 M(0)} \quad \text{for } D_{sp} \text{ in } (\text{Jm}^2) \quad \text{or one can write } k_1 = \frac{0.0346}{\mu_0 M(0)} \quad \text{for } D_{sp} \text{ in } (\text{meV}\text{\AA}^2)$$

This gives for the magnetostriction behavior in the low temperature range:

$$\frac{\lambda_s(T)}{\lambda_s(0)} \propto 1 - n \frac{0.1174\mu_B k_B^{3/2}}{\mu_0 M(0)} \left(\frac{T}{D_{sp}}\right)^{3/2}$$

Taking into account that for the spin wave stiffness constant the unit  $\text{meV}\text{\AA}^2$  is often used one finally obtains the following formula (with  $n=3$  for one-ion and  $n=2$  for two-ion behavior):

$$\lambda_s(T) = \lambda_s(0) \left( 1 - n \frac{0.0346}{\mu_0 M(0)} \left(\frac{T}{D_{sp}}\right)^{3/2} \right) \quad (8)$$

As in Fig. 13 shown the measurements agrees very well with the theoretical predictions (factor  $n$  is between 2 and 3). For samples of the composition  $\text{Fe}_{85-x}\text{Co}_x\text{B}_{15}$  in general a decreasing factor  $n$  with decreasing magnetization  $M(0)$  was found, which corresponds to an increasing Co-content. Therefore as an approximation which is valid for many samples one obtains:

$$\lambda_s(T) = \lambda_s(0) \left( 1 - k \cdot \left(\frac{T}{D_{sp}}\right)^{3/2} \right) \quad (9)$$

with a factor  $k$  between 0.045 and 0.058.  $T$  is the temperature in Kelvin,  $D_{sp}$  is the spin wave stiffness constant in  $\text{meV}\text{\AA}^2$ . Other experimental examples as e.g. measurements on pure  $\text{Fe}_{85}\text{B}_{15}$  and on Finemet can be found in [13,15].

## Acknowledgement

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